

Hong Kong Mathematics Olympiad (2004 – 2005)

Heat Event (Group)

香港數學競賽 (2004 – 2005)

初賽項目 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若 $x = \frac{19}{97} + \frac{19}{97} \times 2 + \frac{19}{97} \times 3 + \cdots + \frac{19}{97} \times 10$ 及 a 是最接近 x 的整數，求 a 的值。

Let $x = \frac{19}{97} + \frac{19}{97} \times 2 + \frac{19}{97} \times 3 + \cdots + \frac{19}{97} \times 10$ and a is the integer that is the closest to x , find the value of a .

2. 已知正方形 $ABCD$ 的面積是 130 cm^2 及圓 O 經過點 A 、 B 、 C 及 D 。若圓 O 的面積是 $b \text{ cm}^2$ ，求 b 的值。(取 $\pi = 3.14$)

Given that the area of a square $ABCD$ is equal to 130 cm^2 and a circle O passes through the points A , B , C and D . If the area of the circle O is $b \text{ cm}^2$, find the value of b . (take $\pi = 3.14$)

3. 已知 p 、 q 和 r 是方程 $x^3 - x^2 + x - 2 = 0$ 的三個不同的根。若 $Q = p^3 + q^3 + r^3$ ，求 Q 的值。

Given that p , q and r are distinct roots of the equation $x^3 - x^2 + x - 2 = 0$. If $Q = p^3 + q^3 + r^3$, find the value of Q .

4. 當一個三位數減去它的各個數位的數字的和，其差還是一個三位數 $\overline{46x}$ ，求 x 的值。

When a 3-digit number minus the sum of the values of the three digits, the difference is a 3-digit number $\overline{46x}$, find the value of x .

5. 若 B 是整數且 $B > (\sqrt{2} + \sqrt{3})^6$ ，求 B 最小可能的值。

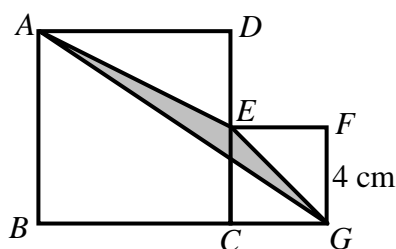
If B is an integer and $B > (\sqrt{2} + \sqrt{3})^6$, find the smallest possible value of B .

6. 若一正八面體的邊長為 1 cm ，其體積為 $f\text{ cm}^3$ ，求 f 的值。

Suppose the side of a regular octahedron is equal to 1 cm and the volume is equal to $f\text{ cm}^3$, find the value of f .

7. 如圖一， $ABCD$ 和 $CEFG$ 是兩個正方形， $FG = 4\text{ cm}$ 。若 $\triangle AEG$ 的面積是 $g\text{ cm}^2$ ，求 g 的值。

In Figure 1, $ABCD$ and $CEFG$ are two squares and $FG = 4\text{ cm}$. If the area of $\triangle AEG$ is equal to $g\text{ cm}^2$, find the value of g .



圖一

Figure 1

8. 設 x 為實數。若 h 是 x 的最大值使得 $2(\log_{\frac{1}{2}} x)^2 + 9\log_{\frac{1}{2}} x + 9 \leq 0$ ，求 h 的值。

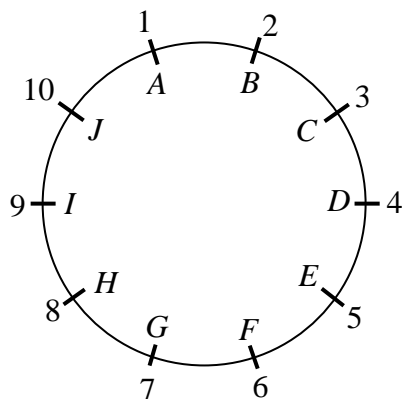
Let x be a real number. If h is the greatest value of x such that $2(\log_{\frac{1}{2}} x)^2 + 9\log_{\frac{1}{2}} x + 9 \leq 0$, find the value of h .

9. 已知在三角形 ABC 內的一點 O 到三角形三邊的垂線的長度均為 2 cm ，而 $\triangle ABC$ 的周界為 21 cm 。若 $\triangle ABC$ 的面積是 $k\text{ cm}^2$ ，求 k 的值。

Given that the perpendicular distances from the point O to three sides of a triangle ABC are all equal to 2 cm and the perimeter of $\triangle ABC$ is equal to 21 cm . If the area of $\triangle ABC$ is equal to $k\text{ cm}^2$, find the value of k .

10. 如圖二，十人圍成一圈，並依座號 $1, 2, 3, \dots, 10$ 而坐。每人選擇一個整數，分別是 A, B, C, \dots, J ，並將這個數字告訴他左右兩個鄰座的人。每人跟着算出他左右兩個鄰座所選的數的算術平均數。若各人所算出的平均數與其座號相等，求 F 的值。

In Figure 2, ten people are sitting in a round table with sitting numbers $1, 2, 3, \dots, 10$ respectively. Each of them chooses an integer A, B, C, \dots, J respectively and tells the people on his left and right about his chosen number. Then each of them calculates the average number of the chosen numbers of his two neighborhoods and announces this average number. If all the announced average numbers are the same as the corresponding sitting numbers, find the value of F .



圖二

Figure 2